SOME MULTIPLE INTEGRAL RELATIONS INVOLVING GENERAL
CLASS OF POLYNOMIALS AND I-FUNCTION

Naseem Ahmad Khan[*]
Yashwant Singh[**]

Abstract (10pt)

In the present paper.we obtain a finite integral involving general class of polynomials and I-function. Next with the application of this and the lamma due to Srivastava et.al. [12], we obtain two general multiple integrals relation involving general class of polynomials. I-function and two arbitrary function f and g. By suitably specializing the functions f and g occuring in the main integral relation, a number of multiple integrals are evaluated which are new and quit general in nature.

Author correspondence:

Naseem Ahmad Khan
Department of Applied Mathematics,
Vivekananada College of Technology & Management
Aligarh-202002, U.P., India

1. Introduction :

The I-function introduced and studied by Saxena occuring in this paper is defined as follows[10];

$$I[z] = I_{p_i,q_i,;r}^{m,n}[z] = I_{p_i,q_i,;r}^{m,n}\left[z \left| \begin{array}{c} (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1,p_i} \\ (b_j, \beta_j)_{1,m}, (b_{ji}, \beta_{ji})_{m+1,q_i} \end{array} \right. \right]$$
$$= \frac{1}{2\pi\omega} \int_L \chi(\xi) z^{\xi} \, d\xi \tag{1.1}$$

where $\omega = \sqrt{-1}$,

$$\chi(\xi) = \frac{\prod\limits_{j=1}^{m} \Gamma(b_j - \beta_j\xi) \prod\limits_{j=1}^{n} \Gamma(1 - a_j + \alpha_j\xi)}{\sum\limits_{i=1}^{r}\{\prod\limits_{j=m+1}^{q_i}\{\Gamma(1 - b_{ji} + \beta_{ji}\xi)\} \prod\limits_{j=n+1}^{p_i} \Gamma(a_{ji} - \alpha_{ji}\xi)\}} \tag{1.2}$$

[*] Department of Applied Mathematics, Vivekananada College of Technology & Management, Aligarh-202001, India.
[**] Department of Mathematics, Government College, Kaladera, Jaipur, Rajasthan, India.

$p_i(i = 1, \cdots, r)$, $q_i(i = 1, \cdots, r)$, m, n are integers satisfying $0 \le n \le p_i$, $0 \le m \le q_i$, $(i = 1, \cdots, r)$, r is finite,$\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji}$ are real and positive and a_j, b_j, a_{ji}, b_{ji} are complex numbers such that

$$\alpha_j(b_h + \nu) \ne \beta_h(a_j - v - k); h = 1, \cdots, m; j = 1, \cdots, n.$$

Srivastava [11]has introduced the general class of polynomial

$$\mathbf{S}_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{m_k}}{k!} \quad A_{n,k} x^k, n = 0, 1, 2, \cdots \tag{1.3}$$

where m is arbitrary positive integer and coefficients$A[n, k(n, k > 0)$ are arbitrary constant, real or complex.

we shall use the following notations;$A^* = (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1,p_i}$; $B^*(b_j, \beta_j)_{1,m}, (b_{ji}, \beta_{ji})_{m+1,q}$

Agarwal and Chaubey [2], (see also Srivastava and Manocha [13], p.447, eq(16)) studied the following general sequence of function:

$$\mathbf{R}_n^{(\alpha,s)}[x; a, b, s, d; p, q; \gamma, \delta; \omega(x)] = \frac{(ax^p + b)^{-\alpha}(sx^q + d)^{-s}}{k_n \omega(x)}$$

$$\times \mathbf{T}_{k,l}^n[(ax^p + b)^{\alpha+\gamma_n}(sx^q + d)^{s+\delta_n}\omega(x)] \tag{1.4}$$

with the differential operator $T_{k,l}$, being defined as

$$\mathbf{T}_{k,l} = x^l(k + xD_x) \quad where \quad D_x = \frac{d}{dx} \tag{1.5}$$

In (1.4), $\{k_n\}$, $(n = 0, 1, \cdots, \infty)$ is a sequence of constants, $\omega(x)$ is independent of n and differentiable an arbitrary number of times. The definition (1.4) was motivated essentially by an earlier work of srivastava and Panda [14] (see also srivastava and Manocha [13]). On taking $\omega(x) = 1$, $p = d = 1$, $s = -\tau$ in (1.4) and replaying s by $\frac{s}{\tau}$ therein we arrive at the following polynomial set after making some obvious changes in parameters (see Gupta et. al. [4]):

$$\mathbf{S}_n^{(\alpha,s,\tau)}[x; r, s, q, A, B, k, l] = (Ax + b)^{-\alpha}(1 - \tau x^l)^{-s/\tau}$$

$$\times \mathbf{T}_{k,l}^n[(Ax + B)^{\alpha+qn}(1 - \tau x^l)^{s/r+sn}] \tag{1.6}$$

The explicit form of the generalized polynomial set given by (1.6) is

$$\mathbf{S}_n^{(\alpha,s,\tau)}[x; r, s, q, A, B, k, l] = B^{q_n} x^{l_n}(1 - \tau x^l)^{sn} l^n \sum_{e,p,u,v} \frac{(-1)^p(-v)_u(-p)_e(-\alpha - qn)_e}{u! v! e! p!}$$

$$\frac{(-s/\tau - sn)_v}{(1 - \alpha - p)_e}((e + k + ru)/l)_n \{\tau x^r/(1 - \tau x^r)\}^v (Ax/B)^p \tag{1.7}$$

where

$$\sum_{e,p,u,v} = \sum_{v=0}^n \sum_{u=0}^v \sum_{p=0}^n \sum_{e=0}^p \tag{1.8}$$

Taking $\tau \to 0$ in (1.7), we get the following

$$\mathbf{S}_n^{\alpha,s,0}[x : r, q, A, B, k, l] = (Ax + b)^{-\alpha} exp(sx^r) \mathbf{T}_{k,l}^n[(Ax + B)^{\alpha+qn} exp(-sx^r)]$$

$$= \sum_{e,p,u,v} \phi(e, p, u, v) x^L \tag{1.9}$$

where

$$\phi(e, p, u, v) = B^{qn-l}l^n \frac{(-1)^p(-v)_u(-p)_e(\alpha)_p}{u!v!e!p!} \frac{(-\alpha - qn)_l}{(1-\alpha-p)_e}\left(\frac{(e+k+ru)}{l}\right)_n A^p B^q \quad (1.10)$$

and

$$L = ln + p + rv, \quad (p, v = 0, 1, \cdots, n) \quad (1.11)$$

It may be pointed out here that through the polynomial set defined by (1.6) is a special case of the general sequence of functions (1.4), yet this polynomial set is sufficiently general in nature and it unifies and extends a number of classical polynomials introduced and studied by various research workers such as Chatterjea [3], Gould and Hopper [5], Krall and Frink [7], Singh and Srivastava [15] etc. Some of special cases of (1.6) are given by Saigo, Goyal and Saxena [9] and Agarwal, Pareek and Saigo [1]. Moreover, the explicit series form similar to (1.7) may not be easily obtainable for the sequence of functions defined by (1.4).

2. Results Required :

The following results will be required in establishing our main integral relations:

(i) Lemma (Srivastava et.al. [12]): Let the function $f(x)$ and $g(x)$ be integrable over the semi infinite integral $(0, \infty)$ and define

$$F(R) = \int_0^{\pi/2} h(R, \theta)d\theta \quad (2.1)$$

where $h(R, \theta)$ is an integrable function of two variables in the rectangular region $0 \leq R < \infty$, $0 \leq \theta \leq \pi/2$. Then

$$\int_0^\infty \int_0^\infty f(x^2+y^2)h\{(x^2+y^2)^{1/2}, tan^{-1}(y/x)\}dxdy = \frac{1}{2}\int_0^\infty f(t)F(\sqrt{t})dt \quad (2.2)$$

and

$$\int_0^\infty \int_0^\infty \int_0^\infty (x^2+y^2)^{-1/2}f(x^2+y^2+z^2)g[tan^{-1}\{(x^2+y^2)^{1/2}/z\}]h\{(x^2+y^2+z^2)^{1/2}tan^{-1}(y/x)\}dxdydz$$

$$= \int_0^\infty \int_0^\infty f(u^2+v^2)g[tan^{-1}(u/v)]F[(u^2+v^2)^{1/2}]dudv \quad (2.3)$$

Provided that the various integral involved are absolutely convergent.

(ii) A result due to Kalla et.al. [6]:

$$H'[at, bt] = \mathbf{I}_{p_1'q_1' \ : p_2',q_2':p_3',q_3':r}^{0,n_1' \ ;1,n_2';1,n_3'}\left[\begin{array}{c} at \\ : \\ bt \end{array} \middle| \begin{array}{c} (a_j'; A_j', A_j'')_{1,p'}, (e_j, E_j)_{1,p_1'}, (g_i, G_i)_{1,p_1'} \\ (b_j'; B_j', B_j'')_{1,q'}, (0,1), (f_j, F_j)_{1,q_2'}, (0,1), (h_j, H_j)_{1,q_3'} \end{array}\right]$$

$$= \sum_{M'=0}^\infty \phi(M')\frac{(-at)^{M'}}{M'!} \quad (2.4)$$

where

$$\phi(M') = \sum_{N'=0}^{M'} \phi'(M'-N', N')\theta_1'(M'-N')\theta_2'(N')(b/a)^{N'}\binom{M'}{N'} \quad (2.5)$$

3. A Useful Integral :

We obtain the following integral, which will be required in the next section:

$$\int\limits_{0}^{\pi/2} e^{i(\alpha+\beta)\theta}(sin\theta)^{\alpha-1}(cos\theta)^{\beta-1}e^{-\mu R}S_n^m[y_1R_1^{2\omega'}e^{i(\lambda+\omega)\theta}(sin\theta)^{\lambda}(cos\theta)^{\omega}]S_n^{\alpha,\beta,0}[y_2R^{2\rho}e^{i(\zeta+\eta)\theta}$$

$$(sin\theta)^{\zeta}(cos\theta)^{\eta}:r',q,A,B,k,l]\mathbf{H}'[ae^{i(\sigma+\rho)\theta}(sin\theta)^{\sigma}(cos\theta)^{\rho},be^{i(\sigma+\rho)\theta}(sin\theta)^{\sigma}(cos\theta)^{\rho}]$$

$$\mathbf{I}[gR^{2\rho_1}e^{i(\sigma_1+\delta_1)\theta}(sin\theta)^{\sigma_1}(cos\theta)^{\delta_1}]d\theta = \sum_{k=0}^{[\frac{n}{m}]}\sum_{M'=0}^{\infty}\sum_{i,\delta,t',p}\frac{(-n)_{mk}}{k!M'!}y_1^ky_2^LR^{z(\omega'k+L)}$$

$$e^{-\mu R}(-a)^{M'}\phi(i,\delta,t',p)\phi(M')A_{n,k}\ exp[\frac{i\pi(\alpha+\lambda k+\zeta L+\sigma M'}{2}]$$

$$I_{p_i+2,q_i+2;r}^{m,n+2}\left[zR^{2\rho_1}exp(i\pi\sigma_1/2)\ \middle|\ \begin{array}{l}(1-\alpha-\lambda k-\zeta L-\sigma M',\sigma_1)\\[1em]B^*\end{array}\right.$$

$$(1-\beta-\omega k-\eta L-\rho M',\delta_1),\quad A^*$$

$$\left.(1-\alpha-(\lambda+\omega)k-(\zeta+\eta)L-(\sigma+\rho)M',\sigma_1+\delta_1)\ \right] \tag{3.1}$$

The integral (3.1)is valid under the following set of conditions:

(i) $\sigma_1 > 0, \delta_1 > 0, Re(\alpha) > 0, Re(\beta) > 0$

(ii) $[Re(\alpha) + \sigma_1\min\limits_{1\le j\le m}Re(b_j/\beta_j)] > 0, [Re(\beta) + \delta_1\min\limits_{1\le j\le m}Re(b_j/\beta_j)] > 0.$

(iii) m is an arbitrary positive integer and the coefficients $n,k(n,k\ge 0)$ are arbitrary constants, real or complex.

(iv) The series occuring on the right hand side of (3.1) is absolutely convergent.

Proof:

To prove (3.1), we first express the general classes of polynomials and the I-function of two variable occurring on the left hand side of (3.1) in series form given by (1.3), (1.8) and (2.4) interchange the orders of summations and integration (which is permissible under the conditions stated). Now, we express the I-function in Mellin-Barnes contour representation given by (1.1), interchange the orders of θ and ξ, integrals and evaluate the θ integral with the help of the following known result Mac Robert [8]).

$$\int\limits_{0}^{\pi/2} e^{i(\alpha+\beta)\theta}(sin\theta)^{\alpha-1}(cos\theta)^{\beta-1}d\theta = exp(i\pi\alpha/2)\frac{\Gamma\alpha\Gamma\beta}{\Gamma\alpha+\beta} \tag{3.2}$$

Finally we interpret the result thus obtained with the help of (1.1) and easily arrive at the desired result (3.1)

4. The Main Integral

The following double and triple integral relations will he established in this section.

$$\int_0^\infty \int_0^\infty x^{\beta-1} y^{\alpha-1} (x^2+y^2)^{1-(\alpha+\beta)/2} exp\{i(\alpha+\beta)tan^{-1}\left(\frac{y}{x}\right) - \mu(x^2+y^2)^{1/2}\} f(x^2+y^2)$$

$$.\mathbf{S}_n^m[y_1 \; exp\{i(\lambda+\omega)tan^{-1}\left(\frac{y}{x}\right)\} y^\lambda x^\omega (x^2+y^2)^{\omega'-(\omega+\lambda)/2}]$$

$$.\mathbf{S}_n^{\alpha,\beta,0}[y_2 \; exp\{i(\zeta+\eta)tan^{-1}\left(\frac{y}{x}\right)\} y^\zeta x^\eta (x^2+y^2)^{\rho-(\zeta+\eta)/2}; r', q, A, B, k, l]$$

$$.\mathbf{H}'[a \; exp\{i(\sigma+\rho)tan^{-1}\left(\frac{y}{x}\right)\} y^\sigma x^\rho (x^2+y^2)^{-(\sigma+\rho)/2}, b \; exp\{i(\sigma+\rho)tan^{-1}\left(\frac{y}{x}\right)\} y^\sigma x^\rho (x^2+y^2)^{-(\sigma+\rho)/2}]$$

$$.\mathbf{I}[z_1 \; exp\{i(\sigma_1+\delta_1)tan^{-1}\left(\frac{y}{x}\right)\} \; x^{\delta_1} y^{\sigma_1}(x^2+y^2)^{\rho_1-(\sigma_1+\delta_1)/2}]$$

$$= \frac{1}{2}\sum_{k=0}^{[\frac{n}{m}]} \sum_{M'=0}^\infty \sum_{i,\delta,\alpha,p} \frac{(-n)_{mk}}{k!M'!} y_1^k y_2^L (-a)^{M'} \phi(i,\delta,t',p)\phi(M')A_{n,k} \; exp\left\{\frac{i\pi(\alpha+\lambda k+\zeta L+\sigma M'}{2}\right\}$$

$$\int_0^\infty t^{\omega'k+\rho L}e^{-\mu v t}I_{p_i+2,q_i+2;r}^{m,n+2}\left[\begin{array}{c|c} z_1 \; t^{\rho_1}exp(i\pi\sigma_1/2) & (1-\alpha-\lambda k-\zeta L-\sigma M', \sigma_1) \\ & B^* \end{array}\right.$$

$$\left.\begin{array}{c} (1-\beta-\omega k-\eta L-\rho M : \delta_1)A^* \\ (1-\alpha-\beta-(\lambda+\omega)k-(\zeta+\eta)L-(\sigma+\rho)M', \sigma_1+\delta_1) \end{array}\right] f(t)dt \qquad (4.1)$$

and

$$\int_0^\infty \int_0^\infty \int_0^\infty x^{\beta-1} y^{\alpha-1} (x^2+y^2)^{(1-\alpha+\beta)/2} exp\{i(\alpha+\beta)tan^{-1}\left(\frac{y}{x}\right) - \mu(x^2+y^2+z^2)^{1/2}\} f(x^2+y^2+z^2)$$

$$.g[tan^{-1}\frac{(x^2+y^2)^{1/2}}{z}]\mathbf{S}_n^m[y_1 \; (x^2+y^2+z^2)^{\omega'} exp\{i(\lambda+\omega)tan^{-1}\left(\frac{y}{x}\right)\} y^\lambda x^\omega (x^2+y^2)^{-(\lambda+\omega)/2}$$

$$.\mathbf{S}_n^{\alpha,\beta,0}[y_2 \; (x^2+y^2+z^2)^\rho \; exp\{i(\zeta+\eta)tan^{-1}\left(\frac{y}{x}\right)\} y^\zeta x^\eta (x^2+y^2)^{-(\zeta+\eta)/2}; r', q, A, B, k, l]$$

$$.\mathbf{H}'[a \; exp\{i(\sigma+\rho)tan^{-1}\left(\frac{y}{x}\right)\} y^\sigma x^\rho (x^2+y^2)^{-(\sigma+\rho)/2}, b \; exp\{i(\sigma+\rho)tan^{-1}\left(\frac{y}{x}\right)\} y^\sigma x^\rho (x^2+y^2)^{-(\sigma+\rho)/2}]$$

$$.\mathbf{I}[z_1 \; (x^2+y^2+z^2)^{\rho_1}exp\{i(\sigma_1+\delta_1)tan^{-1}\left(\frac{y}{x}\right)\} \; x^{\delta_1}y^{\sigma_1}(x^2+y^2)^{-(\sigma_1+\delta_1)/2}]$$

$$= \frac{1}{2}\sum_{k=0}^{[\frac{n}{m}]} \sum_{M'=0}^\infty \sum_{i,\delta,\alpha,p} \frac{(-n)_{mk}}{k!M'!} y_1^k y_2^L (-a)^{M'} \phi(i,\delta,t',p)\phi(M')A_{n,k} \; exp\left\{\frac{i\pi(\alpha+\lambda k+\zeta L+\sigma M'}{2}\right\}$$

$$\int_0^\infty \int_0^\infty f(u^2+v^2)g\{tan^{-1}\left(\frac{u}{v}\right)\}(u^2+v^2)^{\omega'k+\rho L} \; exp\{-\mu(u^2+v^2)\}$$

$$I_{p_i+2,q_i+1;r}^{m,n+2}\left[\begin{array}{c|c} z_1 \; (u^2+v^2)^{\rho_1}exp(i\pi\sigma_1/2) & (1-\alpha-\lambda k-\zeta L-\sigma M', \sigma_1) \\ & B^* \end{array}\right.$$

$$\left.\begin{array}{c} (1-\beta-\omega k-\eta L-\rho M', \delta_1) \; A^* \\ (1-\alpha-\beta-(\lambda+\omega)k-(\zeta+\eta)L-(\sigma+\rho)M', \sigma_1+\delta_1) \end{array}\right] du \; dv \quad (4.2)$$

where sets of conditions (1), (ii), (iii) and (iv) mentioned with (3.1) are satisfied.

Proof :

To establish the integral relations (4.1) and (4.2) we take in (2.1)

$$\int_0^{\pi/2} h(R,\theta)d\theta = L.H.S. of (3.1) \tag{4.3}$$

we obtain $F(R) = $ R.H.S. of (3.1)

Now substituting the values of $h(R,\theta)$ and $F(R)$ in (2.2) and (2.3) in succession, we obtain the integral relations (4.1) and (4.2) after little simplification.

5. Special Cases :

The integral relations (4.1) and (4.2) are quite general in nature on account of the arbitrary nature of the functions f and g and also on account of the presence of H-function of two variables, the I-function and general classes of polynomials. A very large number of (known and new) integrals can be derived as special cases. We mention below a special case of our result.

If we set $\omega = \lambda = \zeta = \eta = 0$ and max $(\sigma_1, \delta_1) = 0$ in (4.1),we get

$$\int_0^\infty \int_0^\infty x^{\beta-1}y^{\alpha-1}(x^2+y^2)^{1-(\alpha+\beta)/2}exp\{i(\alpha+\beta)tan^{-1}\left(\frac{y}{x}\right)-\mu(x^2+y^2)^{1/2}\}f(x^2+y^2)\mathbf{S}_n^m[y_1\,(x^2+y^2)^{\omega'}$$

$$\mathbf{S}_n^{\alpha,\beta,0}[y_2\,(x^2+y^2)^\rho;r',q,A,B,k,l]\mathbf{H}'[a\,exp\{i(\sigma+\rho)tan^{-1}\left(\frac{y}{x}\right)\}y^\sigma x^\rho(x^2+y^2)^{-(\sigma+\rho)/2},$$

$$b\,exp\{i(\sigma+\rho)tan^{-1}\left(\frac{y}{x}\right)\}y^\sigma x^\rho(x^2+y^2)^{-(\sigma+\rho)/2}]\mathbf{I}[z_1\,(x^2+y^2)^{\rho_1}exp\{(i\pi+\sigma_1)/2]dxdy$$

$$= \frac{1}{2}\sum_{k=0}^{[\frac{n}{m}]}\sum_{M'=0}^\infty\sum_{i,\delta,\alpha,p}\frac{(-n)_{mk}}{k!M'!}y_1^k y_2^L(-a)^{M'}\phi(i,\delta,t',p)\phi(M')A_{n,k}\,exp\,\{\frac{i\pi(\alpha+\omega M')}{2}\}$$

$$\frac{\Gamma(\alpha+\sigma M')\Gamma(\beta+\rho M')}{\Gamma(\alpha+\beta+\sigma M'+\rho M')}\int_0^\infty t^{\omega'k+\rho L}e^{-\mu vt}I[z_1\,t^{\rho_1}exp(i\pi\sigma_1/2)]\,f(t)dt \tag{5.1}$$

If we further put $M = 1, N = 0, K = 0, A_{0,0} = 1$ and $A = 1, B = 0, q = k = 0, l = r' = 1$ and $n = 0$ in (4.1)and (4.2) then both the polynomials reduce to unity and we arrive at a number of results similar to those considered by Srivastava et.al. [12].

6. Applications :

By suitable choosing the functions f and g in the main integral relations, a large number of interesting double and triple integrals can be evaluated. We shall however, obtain here only one double and one triple integral by way of illustration.

Thus if in (4.2), we set

$$g(t) = e^{i(\alpha+\beta)\theta}(sint)^{\alpha-1}(cost)^{\beta-1}H'[ae^{i(\sigma+\rho)t}(sint)^\sigma(cost)^\rho, b\,e^{i(\sigma+\rho)t}(sint)^\sigma(cost)^\rho] \tag{6.1}$$

We arrive at the following integral relation on making use of (5.1)

$$\int_0^\infty \int_0^\infty \int_0^\infty (xz)^{\beta-1}y^{\alpha-1}(x^2+y^2)^{\beta/2}(x^2+y^2+x^2)^{1-(\alpha+\beta)/2}f(x^2+y^2+x^2)exp\{i(\alpha+\beta)tan^{-1}\left(\frac{y}{x}\right)$$

$$+tan^{-1}\left(\frac{(x^2+y^2)^{1/2}}{z}\right)\}\mathbf{S}_n^m[y_1(x^2+y^2+z^2)^{\omega'}\,exp\{i(\lambda+\omega)tan^{-1}\left(\frac{y}{x}\right)\}x^\omega y^\lambda(x^2+y^2)^{-(\lambda+\omega)/2}]$$

$$.\mathbf{S}_n^{\alpha,\beta,0}[y_2(x^2+y^2+z^2)^\rho\,exp\{i(\eta+\zeta)tan^{-1}\left(\frac{y}{x}\right)\}x^\eta y^\zeta(x^2+y^2)^{-(\zeta+\eta)/2};r',q,A,B,k,l]$$

$$.\mathbf{H}'[a\,exp\{i(\eta+\rho)tan^{-1}\left(\tfrac{y}{x}\right)\}x^\rho y^\sigma(x^2+y^2)^{-(\sigma+\rho)/2},\,b\,exp\{i(\sigma+\rho)tan^{-1}\left(\tfrac{y}{x}\right)\}x^\rho y^\sigma(x^2+y^2)^{-(\sigma+\rho)/2}]$$

$$.\mathbf{I}[z_1(x^2+y^2+z^2)^{\rho'}\,exp\{i(\sigma_1+\delta_1)tan^{-1}\left(\tfrac{y}{x}\right)\}\;x^{\delta_1}y^{\sigma_1}(x^2+y^2)^{\rho_1-(\sigma_1+\delta_1)/2}]dx\,dy\,dz$$

$$=\frac{1}{2}\sum_{k=0}^{[\frac{n}{m}]}\sum_{M',M''=0}^{\infty}\sum_{i,\delta,t',p}\frac{(-n)_{mk}}{k!M'!M''!}y_1^k y_2^L(-a)^{M'+M''}\phi(i,\delta,t',p)A_{n,k}\phi(M')\phi(M'')$$

$$.exp\left\{\frac{i\pi(2\alpha+\lambda k+\zeta L+\sigma M'+\sigma M'')}{2}\right\}\frac{\Gamma(\alpha+\sigma M'')\Gamma(\beta+\rho M'')}{\Gamma(\alpha+\beta+\sigma M''+\rho M'')}$$

$$\int_0^{\infty}t^{\omega'k+\rho L}e^{-\mu vt}\bar{H}_{P+2,Q+1}^{M,N+2}\left[\;z_1\,t^{\rho_1}exp(i\pi\sigma_1/2)\;\left|\;\begin{array}{l}(1-\alpha-\lambda k-\zeta L-\sigma M',\sigma_1)\\[6pt]B^*\end{array}\right.\right.$$

$$\begin{array}{l}(1-\beta-\omega k-\eta L-\rho M',\delta_1)\quad A^*\\[6pt]\left.(1-\alpha-\beta-(\lambda+\omega)k-(\zeta+\eta)L-(\sigma+\rho)M',\sigma_1+\delta_1)\end{array}\right]f(t)dt\quad (6.2)$$

The conditions of validity of above result can easily be derived frome those mentioned with(3.1).

Next we take

$$f(t)=H'[ct,dt]\tag{6.3}$$

and evaluate the t-integral occuring on the R.H.S. of (4.1) and (6.2) with the help of Gamma functions and arrive at the following multiple integrals after simplifications:

$$\int_0^{\infty}\int_0^{\infty}x^{\beta-1}y^{\alpha-1}(x^2+y^2)^{1-(\alpha+\beta)/2}exp\{i(\alpha+\beta)tan^{-1}\left(\tfrac{y}{x}\right)-\mu(x^2+y^2)^{1/2}\}$$

$$.\mathbf{S}_n^m[y_1\,exp\{i(\lambda+\omega)tan^{-1}\left(\tfrac{y}{x}\right)\}x^\omega y^\lambda(x^2+y^2)^{\omega'-(\omega+\lambda)/2}]$$

$$.\mathbf{S}_n^{\alpha,\beta,0}[y_2\,exp\{i(\eta+\zeta)tan^{-1}\left(\tfrac{y}{x}\right)\}x^\eta y^\zeta(x^2+y^2)^{\rho-(\zeta+\eta)/2};r',q,A,B,k,l]$$

$$.\mathbf{H}'[a\,exp\{i(\sigma+\rho)tan^{-1}\left(\tfrac{y}{x}\right)\}y^\sigma x^\rho(x^2+y^2)^{-(\sigma+\rho)/2},\,b\,exp\{i(\sigma+\rho)tan^{-1}\left(\tfrac{y}{x}\right)\}y^\sigma x^\rho(x^2+y^2)^{-(\sigma+\rho)/2}]$$

$$.\mathbf{H}'[c\,(x^2+y^2)\;d(x^2+y^2)]\mathbf{I}[z_1\,exp\{i(\sigma_1+\delta_1)tan^{-1}\left(\tfrac{y}{x}\right)\}\;x^{\delta_1}y^{\sigma_1}(x^2+y^2)^{\rho_1(\sigma_1+\delta_1)/2}]dxdy$$

$$=\frac{1}{2}\sum_{k=0}^{[\frac{n}{m}]}\sum_{M',M'''=0}^{\infty}\sum_{i,\delta,\alpha,p}\frac{(-n)_{mk}}{k!M'!M'''!}y_1^{k_1}y_2^L(-a)^{M'}\phi(i,\delta,t',p)\phi(M')\phi(M''')A_{n,k}(-a)^{M'}(-c)^{M'''}$$

$$(2/\mu^{2\omega_1k+2\rho L+2M'''+2})\,exp\left\{\frac{i\pi(\alpha+\lambda k+\zeta L+\sigma M')}{2}\right\}$$

$$I_{p_i+3,q_i+1}^{m,n+3}\left[\;z_1\,\mu^{-2\rho_1}exp(i\pi\sigma_1/2)\;\left|\;\begin{array}{l}(1-\alpha-\lambda k-\zeta L-\omega M',\sigma_1),\\[6pt]B^*\end{array}\right.\right.$$

$$\begin{array}{l}(1-\beta-\omega k-\eta L-\rho M',\sigma_1),(1-2\omega'k-2\rho L-2M''',2\rho_1),\quad A^*\\[6pt]\left.(1-\alpha-\beta-(\lambda+\omega)k-(\zeta+\eta)L-(\sigma+\rho)M',\sigma_1+\delta_1)\end{array}\right]\tag{6.4}$$

References

[1] Agarwal, R.K.,Pareek,R.S.,and Saigo,M.A., *A general fractional integral formula*.J. Factional calculus,7(1995), 55-60.

[2] Agarwal, B.D.and Chaubey,J.P., *Operational derivation of generating relations for generalized polynomials*,Indian J. Pure Appl. Math., **11**(1980),1151-1157, ibid, **11**(1981)357-379.

[3] Chatterjee, S.K., *Quelques fonctions generatrices des polynomes d'Hermite,du Point de vue de I'algebra de Lie*, C.R. Acad. Sci. Paris Ser ., **A-B 268**(1969), A 600-A602.

[4] Gupta, K.C.,Goyal, S.P. and SalimT.O., *On theorems connecting the Laplace transform and a generalized fractional integral operator*, Tamkang J. Math, (1998), 323-333.

[5] Gould, H.W. and Hooper,A.T., *Operational formulas connected with two generalizations of Hermite polynomials*., Duke.Math. J., (1962), 51-63

[6] Kalla,S.L. Goyal,S.P. and Agarwal,R.K., *On multiple integral transformation*, Math. Notes. 28,(1981) 15-27

[7] Krall,H.L. and Frink,D., *A new class of orthogonal polynomials :the Bessel polynomial*., Trans. Amer. Math. Soc.**65**(1949),100-115.

[8] Mac Robert T.M., *Beta Function formulas and integrals involving E-function.*, Math.Notes,28 (1981),440-452.

[9] Saigo,M.,Goyal,S.P. ,and Saxena,S. R.P. and Srivastava, K.N., *A theorem relating a generalized Weyl fractional integral, Laplace and Verm transforms with applications*,, J.Fractional Calculus,, **13** (1988),43-56.

[10] Saxena,V.P., *Formal solution of certain new pair of dual integral equations involving H-function*, Cour Nat. Acad.Sci. India 52(A) (1982)366-375

[11] Srivastava, H.M., *A contour integral involving Fox's H-function* India J. Math. **14** (1972), 1-6.

[12] Srivastava, H.M.,Goyal S.P. and Agarwal,R.K.: *Some multiple integral relations for the H-function of several vriables* Bull Inst Mth cad. Sinica, (1981),pp 261-277.

[13] Srivastava, H.M. and Manocha, H.L., *A Treatise on Generating Function* John Wiley and Sons (Halsted Press), New York, Ellis Horwood Chichester (1984).

[14] Srivastava, H.M.and Panda,R., *On the unified presentation of certain classical polynomials*., Bull.Un.Math 12 (4),(1971) 306-314

[15] Singh,R.P. and Srivastava, K.N., *A note on generalization of Laguerre and Humber polynomials*., Rieerca (Napoli), **(2)14**(1963),11-21 Errata,ibid,**15** (1964) maggio-agorto,63.